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Transversely Polarized Antiprotons**

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# Single-Spin Asymmetries in Inclusive Charged Pion Production by Transversely Polarized Antiprotons

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# Abstract

The analyzing power  $A_N$  in inclusive  $\pi^-$  and  $\pi^+$  production has been measured with a 200 GeV/c transversely polarized antiproton beam over a wide  $x_F$  range ( $0.2 \leq x_F \leq 0.9$ ) and at moderate  $p_T$  ( $0.2 \leq p_T \leq 1.5$  GeV/c). The asymmetry  $A_N$  increases with increasing  $x_F$  from zero to large positive values for  $\pi^-$ 's, and decreases from zero to large negative values for  $\pi^+$ 's. A threshold for the onset of the asymmetry is observed around  $p_T \sim 0.5$  GeV/c below which  $A_N$  is essentially zero, and above which  $A_N$  increases (decreases) with  $p_T$  for  $\pi^-$ 's ( $\pi^+$ 's) in the covered  $p_T$  range.

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For the first time a high energy polarized antiproton beam was obtained at Fermilab from the parity violating decay of antilambda hyperons [1]. Inclusive reactions measured with this beam give insight into the spin dependence of the underlying partonic processes and add new input regarding the debated question of the spin structure of polarized protons. The results from polarized lepton deep inelastic scattering [2] suggest that the overall contribution of constituent quarks to the proton helicity is small, thus implying an appreciable contribution either of sea quarks, or of gluons, or possibly of orbital angular momentum to the proton spin structure. Significant polarization effects are known to exist at medium and high energies in meson and hyperon production with hadron beams [3]: pions produced by polarized protons show large values of the analyzing power  $A_N$  at high  $x_F$ , and hyperons produced at high  $x_F$  show large transverse polarization.  $A_N$  in inclusive pion production with polarized protons was also measured at 200 GeV/c [4,5]: the  $\pi^\pm$  asymmetry shows an almost mirror symmetric dependence in  $x_F$ , where  $A_N$  increases with increasing  $x_F$  to large positive values for  $\pi^+$  and decreases with increasing  $x_F$  to large negative values for  $\pi^-$ . More recently, large negative values of  $A_N$  in inclusive  $\Lambda^0$  production at 200 GeV/c and large  $x_F$  have also been published [6]. These effects appear already at relatively low values of the transverse momentum  $p_T$  ( $p_T \sim 0.7 - 1.0$  GeV/c), where Perturbative QCD is not expected to be directly applicable. Models were developed to explain and possibly correlate the spin observables in these processes using static SU(6) wave functions and spin dependent asymmetries introduced into the quark and di-quark production and scattering amplitudes [7]. The features of the pion data are compatible with these models, based on the idea that leading valence quarks remember their polarization in the parent proton, and consistent with the interpretation of  $\Lambda^0$  polarization, where quarks produced in fragmentation processes acquire a transverse polarization.

In this letter we report on the measurement of the analyzing power  $A_N$  in inclusive  $\pi^-$  and  $\pi^+$  production

$$\bar{p} \uparrow + p \rightarrow \pi^- (\pi^+) + X$$

using the 200 GeV/c Fermilab polarized antiproton beam incident on a 1.0 meter long liquid hydrogen target. The kinematic range covered is  $0.2 \leq x_F \leq 0.9$  and  $0.2 \leq p_T \leq 1.5$  GeV/c.  $A_N$  measures the left-right scattering asymmetry with respect to the beam polarization directed normal to the production plane, and positive  $A_N$  corresponds to a larger cross section for particle production to the beam left for beam polarization directed upward.

The polarized antiproton beam was obtained by selecting antiprotons from the weak decay of anti- $\Lambda^0$  particles produced in a primary target by the 800 GeV/c Tevatron extracted proton beam. The design and performance of the beam are described in Ref. [1]. Decay antiprotons are longitudinally polarized in the anti- $\Lambda^0$  decay rest frame; those emitted near  $\pm 90^\circ$  acquire a transverse polarization of  $\mp 64\%$  when Lorentz-boosted to the laboratory frame. The antiproton polarization, on average, was determined by tagging the trajectory in the horizontal plane at an intermediate focus of the beamline, where also the momentum was measured. The tagged polarization interval ranged from  $-0.65$  to  $+0.65$ , thus allowing the simultaneous use of antiprotons of opposite polarization. Events with tagged polarization values from  $0.35$  to  $0.65$  ( $-0.65$  to  $-0.35$ ) were used in the  $A_N$  analysis and defined to have positive (negative) polarization. The average polarization was  $0.45$  for both signs. A spin rotator consisting of 12 dipole magnets rotated the transverse beam polarization from the horizontal to the vertical direction at the experimental target. In order to suppress systematic errors, the polarity of the spin rotator magnets was reversed every 15 Tevatron spills. Typical beam intensities at the target were of the order of  $3 \times 10^6$  particles per 20 second Tevatron spill. About  $17\%$  were antiprotons, the rest were mainly pions from  $K_S^0$  decays and muons, which were vetoed with two beamline Cherenkov threshold counters.

The pions produced at the target were measured with a large forward spectrometer described in Refs. [4,6]. The reconstruction of charged tracks produced at the target was done with two telescopes of multi-wire proportional chamber modules (MWPC), 5 upstream and 5 downstream of a 3-Tm  $\int Bdl$  dipole magnet, which was used for momentum measurement. Each MWPC module had four plane views. Beam particles upstream of the target were tracked with 3 MWPC's. Identification of the charged pions produced at the target was

accomplished with a 25 meter long threshold Cherenkov counter, C1, located downstream of the analyzing magnet. C1 was filled with helium gas and its gas pressure was set so that it would count pions with  $P_\pi \geq 40$  GeV/c ( $x_F \geq 0.2$ ) but not protons or kaons. The  $\pi^-$  and  $\pi^+$  data were taken in separate runs with inverted polarities of the analyzing magnet.

The trigger required antiproton beam definition [1] and pion identification. A simple trigger using three hodoscopes downstream of the analyzing magnet in conjunction with fast programmable electronics selected events where the hodoscope hit patterns were compatible with at least one high momentum ( $P > 40$  GeV/c) trajectory from the target. Charged pions were tagged with C1. Background events due to non-interacting beam particles were reduced by using a beam veto located at the downstream end of the spectrometer.

The reconstruction efficiency of the offline tracking program was found to be better than 85 % for a single track. The position resolution on the production point of a track in the target was better than 1 mm in the transverse plane and a few centimeters along the beam axis. It was required that selected tracks matched the beam track in the target volume within the measured track resolution and that they traversed the C1 fiducial volume determined by its mirrors. These selections were found to be independent of the kinematical variables  $x_F$  and  $p_T$ . The background due to misidentified particles and particle production other than at the target was found to be less than 5 % in the selected sample.

The analyzing power  $A_N$  was determined from the measured yields of pions produced in a well defined azimuthal angular interval around the beam axis using vertically polarized antiprotons of both polarization signs:

$$A_N = \frac{1}{P_B \langle \cos \phi \rangle} \frac{N \uparrow - N \downarrow}{N \uparrow + N \downarrow}.$$

$P_B \langle \cos \phi \rangle$  is the effective beam polarization normal to the production plane and  $\phi$  is the azimuthal angle between the beam polarization direction and the normal to this plane. The selected azimuthal angular interval was  $\pm 45$  degrees from the horizontal plane to the beam right, giving  $\langle \cos \phi \rangle \approx -0.90$ .  $N \uparrow$  ( $N \downarrow$ ) is the number of pions produced for positive (negative) spin orientation of the beam antiprotons at the target, normalized to the corresponding



beam flux.  $N \uparrow$  was obtained by combining events having positive tagged polarization and negative spin rotator polarity with events having negative tagged polarization and positive spin rotator polarity.  $N \downarrow$  was obtained from events with equal tagged polarization sign and spin rotator polarity.

A number of consistency checks were performed to establish that the asymmetry results were free of systematic effects. This asymmetry, to a good accuracy, is independent of detection and reconstruction efficiencies, since pion yields were measured with the same apparatus and both polarizations simultaneously. We evaluated the *false* asymmetries by averaging over opposite spin rotator states or over both tagged polarization signs. The average beam polarization was zero for these sets of events. These asymmetries were found to be consistent with zero (*false*  $A_N = -0.009 \pm 0.012$  for  $\pi^-$  data and  $+0.003 \pm 0.014$  for  $\pi^+$  data), thus indicating no bias in the determination of  $A_N$ .

The analyzing power  $A_N$  is given in Table 1 and shown in Figure 1 as a function of  $x_F$  for the  $\pi^-$  and  $\pi^+$  data over a  $p_T$  range of 0.2–1.5 GeV/c. The data exhibit an almost mirror symmetric dependence in  $x_F$  in which the magnitude of  $A_N$  increases for both  $\pi^-$  and  $\pi^+$  mesons with increasing  $x_F$ , but the sign of  $A_N$  is positive for the  $\pi^-$  data and negative for  $\pi^+$  data. Figure 2 and Table 2 show the same asymmetry as a function of  $p_T$  averaged over the  $x_F$  interval of 0.2–0.9. These data show a threshold effect around  $p_T \sim 0.5$  GeV/c above which  $A_N$  increases in magnitude for both  $\pi^-$ 's and  $\pi^+$ 's (see also Figure 3, where  $A_N$  data are plotted as a function of  $x_F$  for  $p_T \geq 0.5$  GeV/c), and below this  $p_T$  value  $A_N$  is significantly smaller, compatible with zero.

Large values of  $A_N$  have been observed in inclusive pion production experiments with polarized proton beams by E704 [4] and some previous experiments at lower energies [8]. Results were obtained also for  $\pi^0$  production with the same polarized antiproton beam [5] in a  $p_T$  range of 0.5–2.0 GeV/c and similar  $x_F$ . The  $A_N$  results for charged pions presented in this work are compared in Figure 3 with  $\pi^0$  data from Ref. [5] over a similar  $p_T$  range. For  $\pi^0$  data  $A_N$  has the same sign as for  $\pi^-$  data and is about half as large. In  $\bar{p} \uparrow + p \rightarrow \pi^+ + X$   $A_N$  appears to be similar to that in  $p \uparrow + p \rightarrow \pi^- + X$ , while for  $\bar{p} \uparrow + p \rightarrow \pi^- + X$   $A_N$  is

slightly smaller compared to  $p \uparrow + p \rightarrow \pi^+ + X$  data in the same kinematical region.

In summary, the analyzing power for  $\pi^-$  production increases from 0 to about +0.25 with increasing  $x_F$  above  $p_T \sim 0.5$  GeV/c while for  $\pi^+$  production  $A_N$  decreases from 0 to about -0.35 with increasing  $x_F$  above the same  $p_T$ . It appears that  $A_N$  depends primarily on  $x_F$ , and reaches large values above a  $p_T$  threshold of about 0.5 GeV/c.

These results can be explained qualitatively as an effect similar to that proposed to explain the hyperon polarization [7], in which  $q\bar{q}$  pairs produced in fragmentation processes become transversely polarized and at large  $x_F$  the transverse spin of the (anti)protons is correlated to its (anti)quark constituents. To produce a spin-zero meson, the polarized produced  $(q)\bar{q}$  will couple with the spectator (anti)*up* or (anti)*down* quark from the polarized (anti)proton beam only in an antiparallel configuration. The reflected sign of  $\pi^-$ 's with respect to  $\pi^+$ 's (and between  $\bar{p}$  and  $p$  beams) may originate from the fact that the *up* (anti)quark spin is almost fully aligned with that of the (anti)proton for  $x_F$  approaching one, whereas that of the *down* (anti)quark is oppositely aligned. This conclusion is consistent with recent SMC results [9] on semi-inclusive deep inelastic scattering, which probed the polarization of valence quarks in the nucleon. Recent models based on non-perturbative approaches, such as the *soft*  $\pi$  exchange mechanism [10], or resonance-decay interference between real and virtual channels [11], or rotating constituents in the polarized (anti)proton [12], appear to be in good qualitative agreement with the features of the data on the pion production asymmetry measured with both polarized protons and antiprotons.

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# TABLES

TABLE I.  $A_N$  data for  $\bar{p} \uparrow + p \rightarrow \pi^- + X$  and  $\bar{p} \uparrow + p \rightarrow \pi^+ + X$  as a function of  $x_F$ . The reported errors are statistical only.

$x_F$	$A_N(\%)$	$\langle p_T(\text{GeV}/c) \rangle$	No. of events
$\bar{p} \uparrow + p \rightarrow \pi^- + X$			
0.2–0.3	$1.1 \pm 2.6$	0.38	9386
0.3–0.4	$1.4 \pm 2.0$	0.48	14707
0.4–0.5	$5.3 \pm 2.2$	0.51	11688
0.5–0.7	$11.9 \pm 2.5$	0.61	9466
0.7–0.9	$21.8 \pm 6.7$	0.81	1267
$\bar{p} \uparrow + p \rightarrow \pi^+ + X$			
0.2–0.3	$2.0 \pm 2.6$	0.38	8851
0.3–0.4	$1.1 \pm 2.2$	0.46	12167
0.4–0.5	$-6.1 \pm 2.7$	0.55	8339
0.5–0.7	$-14.8 \pm 3.3$	0.66	5264
0.7–0.9	$-34 \pm 11$	0.82	439

TABLE II.  $A_N$  data for  $\bar{p} \uparrow + p \rightarrow \pi^- + X$  and  $\bar{p} \uparrow + p \rightarrow \pi^+ + X$  as a function of  $p_T$ . The reported errors are statistical only.

$p_T(\text{GeV}/c)$	$A_N(\%)$	$\langle x_F \rangle$	No. of events
$\bar{p} \uparrow + p \rightarrow \pi^- + X$			
0.2–0.35	$0.2 \pm 2.0$	0.35	14279
0.35–0.5	$0.4 \pm 2.0$	0.39	14047
0.5–0.7	$8.4 \pm 2.3$	0.44	11630
0.7–1.0	$18.5 \pm 3.1$	0.50	6123
1.0–1.5	$23.5 \pm 7.1$	0.59	1203
$\bar{p} \uparrow + p \rightarrow \pi^+ + X$			
0.2–0.35	$3.4 \pm 2.3$	0.32	10465
0.35–0.5	$1.0 \pm 2.3$	0.36	10676
0.5–0.7	$-6.3 \pm 2.6$	0.41	9101
0.7–1.0	$-18.8 \pm 3.6$	0.47	4672
1.0–1.5	$-27.2 \pm 8.0$	0.55	953

FIGURES

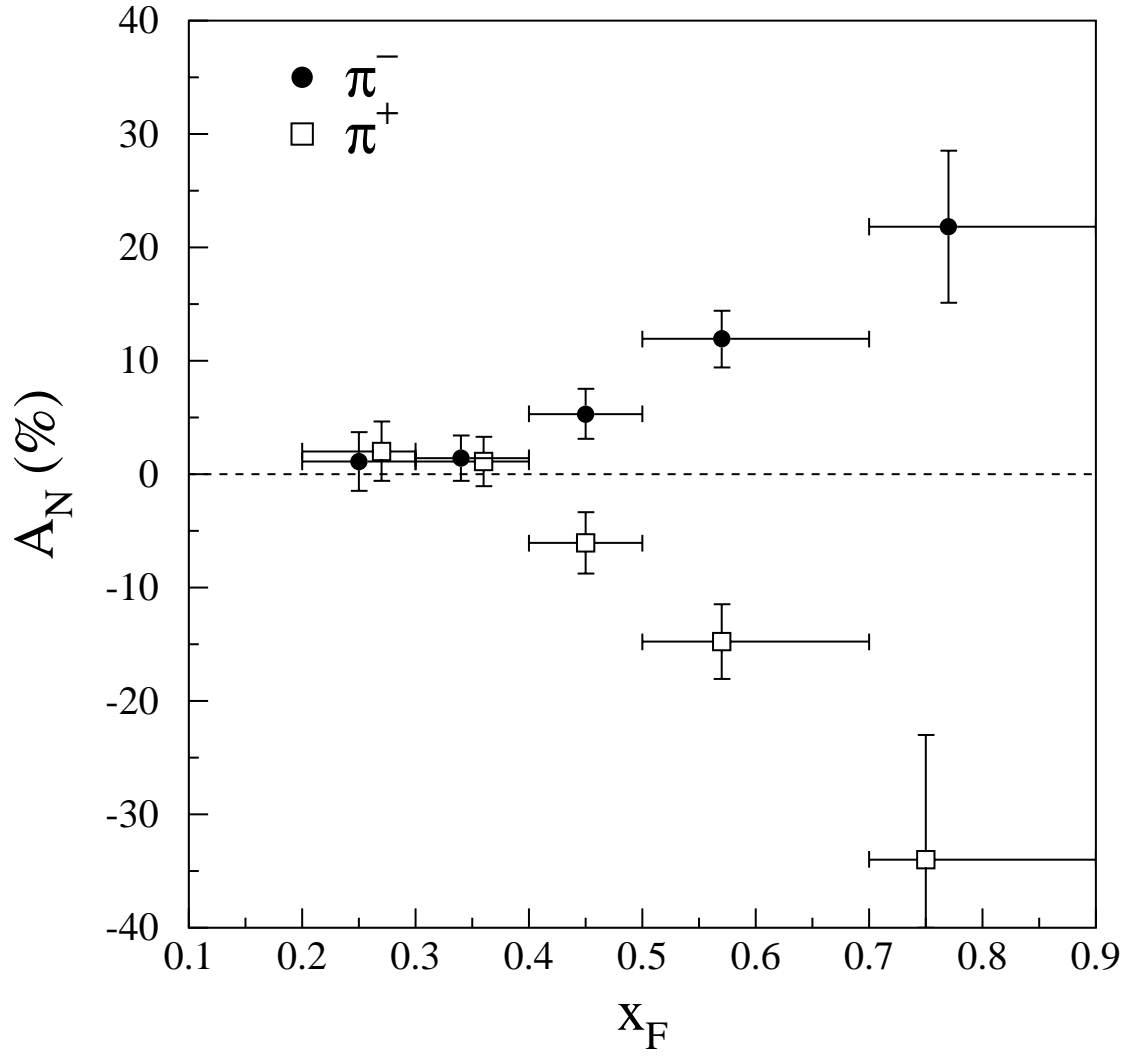


FIG. 1.  $A_N$  data as a function of  $x_F$  for  $\pi^-$  (full circles) and  $\pi^+$  (open squares) integrated over  $p_T$ . For clarity the first two  $\pi^-$  ( $\pi^+$ ) data points are offset by  $-0.01$  ( $+0.01$ ) units in  $x_F$ .

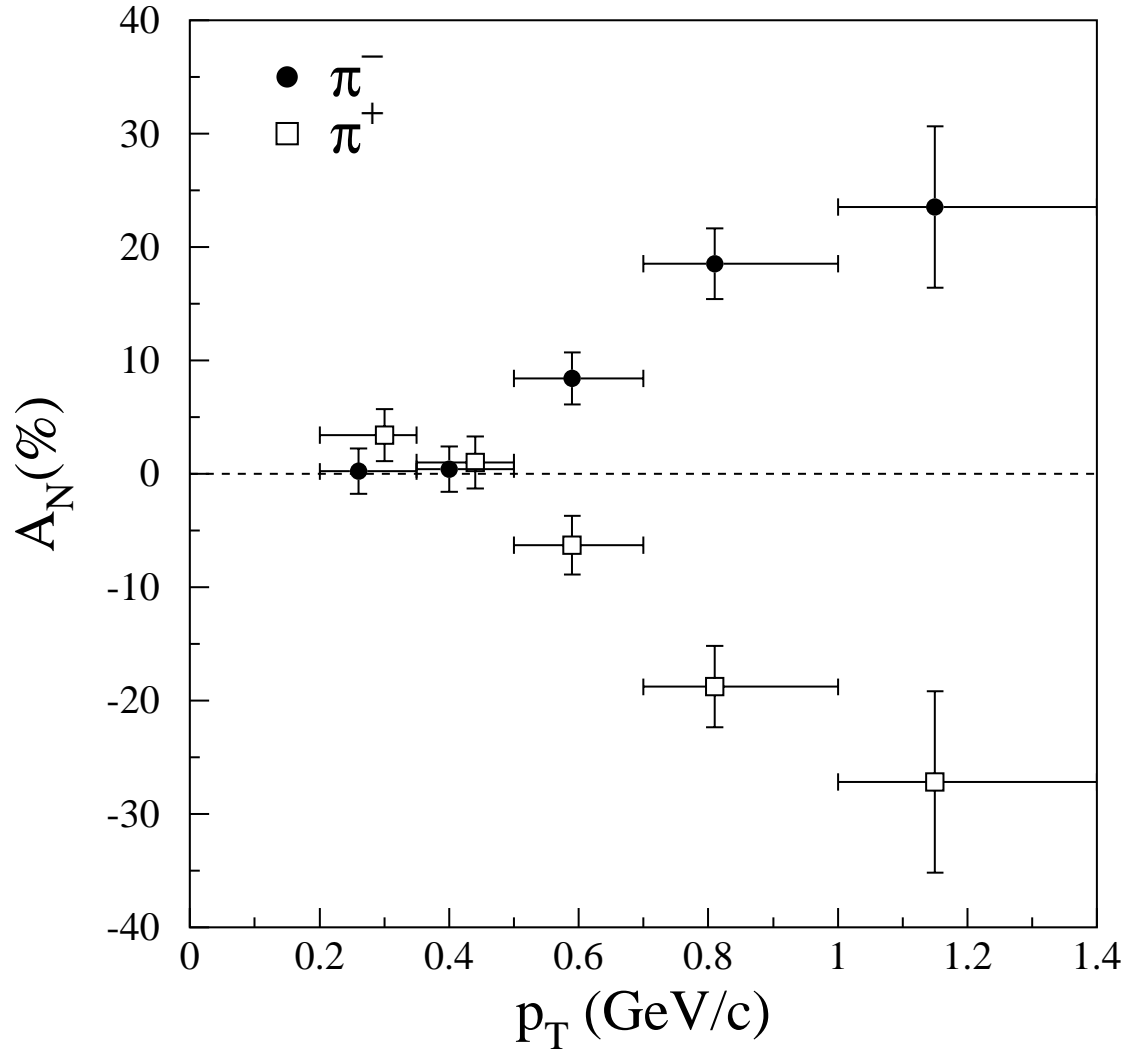


FIG. 2.  $A_N$  data as a function of  $p_T$  for  $\pi^-$  (full circles) and  $\pi^+$  (open squares) in the  $x_F$  range of 0.2–0.9. For clarity the first two  $\pi^-$  ( $\pi^+$ ) data points are offset by  $-.02$  ( $+.02$ ) GeV/c.

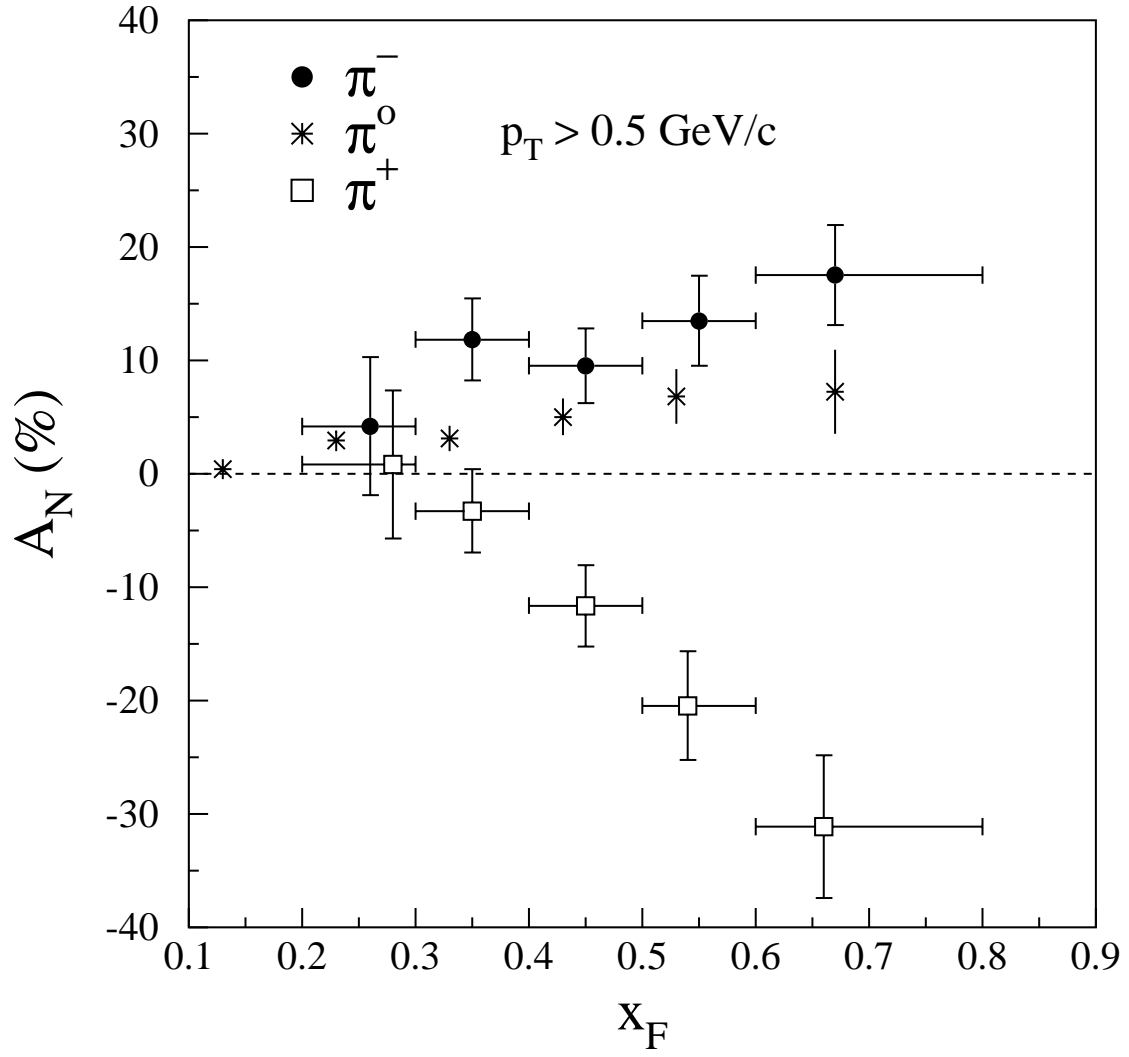


FIG. 3.  $A_N$  data as a function of  $x_F$  for  $\pi^-$  and  $\pi^+$  for  $p_T \geq 0.5 \text{ GeV}/c$ .  $A_N$  data for  $\pi^0$  in a similar  $p_T$  range are also shown [5]. The first  $\pi^-$  and  $\pi^+$  data points are offset by  $-0.01$  and  $+0.01$   $x_F$  units respectively.